Distributed Coverage Control for a Multi-Robot Team in a Non-Convex Environment

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3rd Workshop on Planning, Perception and Navigation for Intelligent Vehicles, 10/11/2009
Outline

1. Introduction
2. Convex Environment
3. Proposed Algorithm
4. Simulations
5. Conclusion
The problem of coverage
Possible tasks

The objective is to achieve an optimal placement for a team of mobile robots in an environment with unknown obstacles

- surveillance of dangerous regions, like areas of chemical, biological or nuclear contamination
- environmental monitoring (air quality, forest fire, ...)
- aiding police during surveillance missions
- ....
Let be $\mathcal{P} = \{p_i\}$ the positions of the $N$ robots and $\mathcal{W} = \{W_i\}$ their regions of competence, with $i = 1, \ldots, N$. We can define a performance function

$$J(\mathcal{P}, \mathcal{W}) = \sum_i \int_{W_i} f(\|q - p_i\|)\phi(q) dq$$

There are two optimization problems:

- Optimal partition $\mathcal{W}$
- Optimal location $\mathcal{P}$

The optimal partition is the Voronoi partition.
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Problem Formulation

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We can define a performance function

$$J(\mathcal{P}, \mathcal{W}) = \sum_i \int_{W_i} f(||q - p_i||)\phi(q) dq$$

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Given an open set $\Omega \subseteq \mathbb{R}^N$, the set $\{V_i\}_{i=1}^k$ is called a tessellation (or partition) of $\Omega$ if $V_i \cap V_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^k V_i = \Omega$. Let $\| \cdot \|$ denote the Euclidean norm on $\mathbb{R}^N$, given a set of points $\{p_i\}_{i=1}^k$ belonging to $\Omega$, the Voronoi region $V_i$ corresponding to the point $p_i$ is defined by

$$V_i = \{q \in \Omega \mid \|q - p_i\| \leq \|q - p_j\| \ \forall j \neq i\}$$

Examples
Optimal Location

\[ \Rightarrow \min_{\mathcal{P}, \mathcal{W}} J(\mathcal{P}, \mathcal{W}) = \min_{\mathcal{P}} J(\mathcal{P}, \mathcal{V}) \]

We have to find the optimal \( \mathcal{P} \)

Condition of stationarity

\[ \nabla J_{\mathcal{V}} = \left[ \cdots \frac{\partial J_{\mathcal{V}}}{\partial p_i} \cdots \right]^T = 0 \quad \text{where} \quad J_{\mathcal{V}} \equiv J(\mathcal{P}, \mathcal{V}) \]

Hence, we have to solve

\[ \frac{\partial J_{\mathcal{V}}}{\partial p_i} = - \int_{V_i} \frac{df(x)}{dx} \left| \frac{q - p_i}{\|q - p_i\|} \right| \phi(q) \, dq = 0 \]
A possible choice for the function \( f(\| \cdot \|) \) is \(^1\)

\[
f(\| q - p_i \|) = \| q - p_i \|^2
\]

The optimal location is the centroidal one

\[
\frac{\partial J_V}{\partial p_i} = 2 \left[ \int_{V_i} (p_i - q) \, dq \right] = 2M_{V_i}(p_i - C_{V_i})
\]

\[
M_V = \int_V \phi(q) \, dq, \quad C_V = \frac{1}{M_V} \int_V q \phi(q) \, dq
\]

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Lloyd Algorithm

The algorithm: calculate the Voronoi partition and the relative centers of mass, or the equivalent points. Move each robot toward the center of mass. Repeat this procedure for each time step until the convergence of the algorithm [Lloyd82].
Let us now restrict our attention to the unweighted problem, i.e. $\phi(q) = 1$. We show the simulations made with 7, 9, 11 robots.
Non-Convex Environment

The Lloyd algorithm can be used only in a convex environment. In the real world, we have to take into account the obstacles. The optimization problem is now too hard to solve.

\[ J(\mathcal{P}) = \int_{\Omega} \min_{p_i} \tilde{d} (q, p_i) \, dq \]

Impossible to solve, especially if the obstacles are unknown.

- Possible approach: repulsive potential field \([Howard \ et \ al.02]\), \([Poduri \ et \ al.04]\)
- Drawback: local minima
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Potential Field + Voronoi Partition

To obtain the coverage of an environment with unknown obstacles, the proposed method combines the Voronoi partition with the repulsive potential field.

The movement of each robot is generated by:
- the repulsion of the other robots
- the repulsion of the closest obstacle
- the attraction of the center of mass of the Voronoi region

Equation of motion

\[ F_{tot} = F_{rep} + F_{att} = m \ddot{q} - \nu \dot{q} \]
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Numerical Simulations

Two different comparisons for the different environments

- Convex
  Comparison with the Lloyd algorithm

- Non-Convex
  Comparison with the repulsive potential field method
The proposed algorithm compared with the Lloyd algorithm in a convex environment

The trajectories are different but the result in terms of cost function is the same
Non-Convex Environment

The proposed algorithm compared with the repulsive potential field in an environment with obstacles

The performance function $J$ has been calculated by a discretization of the environment
Non-Convex Environment

To verify the performance of the proposed method, a comparison between the final cost functions obtained by the two methods has been made for 100 different environments, with the following properties:

- Fixed external boundary
- Number of obstacles randomly varying from 1 to 6, with fixed size and random positions
- Only constraint: obstacles can not form barriers
Non-Convex Environment

Simulations result

- Cost function always lower
- Values less scattered around a mean value
Conclusion

- New algorithm for the coverage of an environment with unknown obstacles
- Same performance than the Lloyd Algorithm in a convex environment
- Better performance than the only repulsive potential field method for a non-convex environment

Outlook
- Extension for a heterogeneous team \(^1\)
- Stochastic optimization methods
- ....

\(^1\)submitted ICRA2010